Chapter 3: Polynomials

When greeted with a power of a power, multiply the two powers. $\mathbf{(x^2)^3 = x^6}$

When multiplying powers with the same base, add the exponents. $\mathbf{15^7 x 15^{14} = 15^{21}}$

When dividing powers with the same base, subtract the exponents. $\mathbf{5^7 \div 5^3 = 5^4}$

When simplifying negative expressions that include exponents: look at your exponent. If the exponent is an odd number, the answer will be negative. If the exponent is even, the answer will be positive. $\mathbf{-4^4 = 256, -4^3 = -64}$

When a term is surrounded by brackets and there is a fraction alongside it, multiply.
$\mathbf{3/5\ (5x^2) = 15/5x^2}$

To calculate the degree of a term, add all the exponents. If a variable does not have an exponent, add an imaginary ^{1}. $\mathbf{b^7c^4d = 12}$

To calculate the degree of a polynomial, calculate all of the terms degrees', the one with the highest value is the degree. $\mathbf{a^2q^4+o+b^3 = 6}$

The literal coefficient/variable is the letter and exponent. $\mathbf{7a^2+3 = a^2}$
The numerical coefficient would be **7** and the constant would be **3**.

There are four different ways to classify polynomials:
1. Monomial = 1 term
2. Binomial = 2 terms
3. Trinomial = 3 terms
4. Polynomial = 4+ terms

When assigning a value to a variable, follow the following format:
Let x (variable) represent the number of (whatever).

A term is separated by a +/- sign.

The **distributive property** is to expand an expressions and multiply all the terms in the brackets by the monomial term thats in front of the bracket.
$\mathbf{5(x+3) = 5x+15}$

Chapter 4: Equations

An equation is a mathematical statement that says two expressions are equal. You solve equations by determining the solution which makes the equation true. **5x4 = 2x10**

Some equations can simply be solved by *inspection*, while some others cannot and require various steps.

If the equation cannot be done through inspection and does not include fractions, you follow this method:
7k = 3k-16 <— Equal signs should be aligned.
7k-3k = -16 <— Rearrange the equation, moving likewise values to the same side, opposite their value (-/+).
4k = -16 <— Divide the number with the variable by the other side's total.
k = -4 <— You're left with the answer, also known as the root.

If you'd like to check your answer, simply swap the variable(s) with your result and calculate each side; they should both equal the same. **7x-4 = -28 : 3x-4-16 = -28**

Next up is multi-step equations, these are practically the same, just with more numbers and variables.
5(x-3) + 3(x-7) = 28 <— Apply the distributive property.
5x-15+3x-21 = 28
8x-36 = 28
8x = 28+36
8x = 64
x = 8

When solving equations involving fractions, the proceeder changes. These are called proportions.

$\frac{-3}{4}(d+3) = \frac{4}{5}(3d-2)$

$\frac{-3d-9}{4} = \frac{12d-8}{5}$

<— Follow distributive property and multiply the contents of the brackets with the numerators, and add the denominators below each set of numbers.

-15d-45 = 48d-32
-15d-48d = -32 + 45
-63d = 13
d = -13/63

<— Cross multiply the fractions, you'll be left with the following. Now, do the exact same steps we learned previously.

Okay, non-proportions are a little different.

$\frac{-3p}{2} + \frac{4}{4} = \frac{7p}{1} + \frac{6}{2}$ <— Find a common denominator of all four fractions.

$\frac{-6p}{4} + \frac{4}{4} = \frac{28p}{4} + \frac{12}{4}$

-6p+4 = 28p+12 <— Convert the fractions into *standard form*. Now, follow the same steps as before.
-6p-28p = 12-4
-34p = 8
p = -4/17

Rearranging equations is all about doing the opposite.
Dividing = Multiplying, Square-rooting = Squaring, Subtracting = Adding and vice versa.
C = 2πr, R = C/2π
S = P/4, P = Sx4

For Algebraic expressions, don't be lazy and be sure to use "Let…" statements that include what it represents and in what (dollars, metres, etc).
Let x represent the time a student had worked, in hours.
3x — 3x2, I worked for six hours.

Chapter 5: Modelling with Graphs

Direct variation occurs when the dependant variable varies by the same factor as the independent variable.

Direct variation can be defined algebraically as **Y/X = K**, where K is the constant of variation.

The graph of a direct variation is a straight line that passes through the origin (0,0).

A partial variation has an equation of the form **Y= MX+B**, where B represents the fixed/initial value of Y and M represents the constant of variation or slope.

The graph of a partial variation is a straight line that does not pass through the origin.

The slope is the steepness of a line segment. The slope is the ratio of the rise and run. It is represented by the variable M.

Slope (M) = Rise/Run or $Y_2 - Y_1 / X_2 - X_1$
(Constant of Variation = Slope)

When the line rises to right, the slope will be positive.
When the line falls to the right, the slope will be negative.
When the slope's denominator is 0, the slope is **undefined**.

The Y intercept of a straight line is simply where the line crosses the Y axis.

Rate of Change: A change in one quality relative to the change in another quantity.
For example: **Slope = -60L/500km**, **Rate of Change = Loss of -60L for every 500km driven. In lowest terms, loss of -0.12L for every km driven.**

Linear vs. Non-Linear

- Equation:
  Y = 3x + 10 **LINEAR**
  Y = x^2 + 10 **NON-LINEAR**
  When the exponent on the variable is one (invisible), then the relation is linear.
  When the exponent is two or more, the relation is non-linear (and/or quadratic).
- Graph:
  If the graph is a straight line, then it's linear. If the line is not straight, then it's non-linear.
- Table of Values:
  Calculate the differences of each set of numbers (y), from bottom to top. If all are constant, it's linear, if not, it's non-linear.

A relation can be represented in four ways:

1. Words
2. Diagram or graph
3. Numbers
4. Equation

Chapter 6: Analyzing Linear Relations

To graph an equation in the form of **y = mx+b**, do the following: plot the “b” on the y-axis, from that point plot the rise and run and keep doing so until you can create a line. For example: **Y = 2x+5 — our points would be: (0,5), (1,7), (2,9)…**

A horizontal line is written in the form **y=b**, where b is the y-intercept. The slope of a horizontal line is zero.
A vertical line on the other hand, is written in the form **x=a**, where a is the x-intercept. The slope of a vertical line is undefined.

Ax+By+C=0 — standard form.
Standard form does not display slope or the y-intercept. We must rearrange the equation (chapter 4) to get those pieces of information.

To graph a line using intercepts (standard form) follow this method:
- Isolate the constant
- Divide the constant by the x and y variable.
- For example: **4x+6y=48** — Divide 48 / 4 and 48 / 6.
  X-Intercept = (12,0). Y-Intercept = (0,8).
- Now plot these two points, and create your line.

Parallel Lines — Lines that run in the same direction and will never intersect.
Perpendicular Lines — Lines that intersect at a right angle.
Coinciding Lines — Lines are the exact same.

With parallel lines, both equations will have the same slope.
With perpendicular lines, both equations will have negative reciprocals.

To find an equation for a line with a slope and a point, simply substitute for y=mx+b, then solve for b.
For example: **Slope of 1/2 and passes through the point (1,5)**
5 = 1/2(1) + b
b = 4.5
Therefore, the equation is y=1/2x + 4.5.

To find an equation of a line given two points, find the slope using the slope formula, and find the y-intercept using substitution. For example: **(1,2), (5,10)**
10 - 2 / 5 - 1 = 8/4 = 2 <— Slope
2 = 1(2) + b
b = 0
Therefore, the equation is 2x.

When finding the point of intersection, either graph the two lines, or follow this method: put both equations in y=mx+b form, have them equal each other and solve for x.
For example: **y=1/2x-3, y =-x-6**
1/2x-3= -x-6
1/2x+x = 3-6
1.5x = -3
x = -2
Now to solve for y, input the x value into either the equations. The answer will be the y.
-(-2)-6 = -4
Therefore, the point of intersection is (-2,-4).

Chapter 2: Relations

Scatterplots help us to see the relationships between variables.

Inference — Conclusion based on reasoning and data.

Independent Variable (X) — A variable that affect the value of another variable.

Dependant Variable (Y) — A variable that is affected by another variable.

Outlier — A measurement that differs significantly from the rest of data; could be an error, or a factor.

Interpolate — Means to estimate a value between two measurements in a set of data.

Extrapolate — Means to estimate a value beyond the range of a set of data.

Linear Relation — An equation that makes a straight line when it is graphed and is a constant.

Non-Linear Relation — An equation that does not make a straight line when graphed and is not constant.

Line of Best Fit — A line on a graph showing the general direction that a group of points seem to be heading.

Curve of Best Fit — A curved line that passes as close as possible to a set of points.

Distance Time Graph — A graph that shows how distance varies with time.

The steeper the line/curve —> The faster the speed.
The flatter the line/curve —> The slower the speed.
Walking away —> Graph rises to the right.
Walking towards —> Graph falls to the right.

Chapter 7: Geometric Relationships

TERMS

Polygon — Closed figure made up of line segments.

Vertex — Point where two or more sides meet.

Interior Angle — Angle formed on the inside of a polygon by two sides meeting at a vertex.

Exterior Angle — Angle formed on the outside of a geometric shape by extending one of the sides past a vertex.

Ray — A part of a line with one endpoint.

Transversal Line — Intersects parallel lines and forms lots of angles and creates the F, C, and Z patterns to help us find missing angles.

Regular Polygon — A polygon with all sides equal and all interior angles equal.

Convex Polygon — A polygon with no part of any line segment joining two points on the polygon outside the polygon.

Concave Polygon — A polygon with parts of some line segments joining two points on the polygon outside the polygon.

Midpoint — The point that divides a line segment into two equal parts.

Median — The line segment joining a vertex of a triangle to the midpoint of the opposite side.

Bisect — Divide into two equal parts.

Right Bisector — A line perpendicular to a line segment and going through its midpoint, 90°.

ANGLE THEORIES

Angle Sum Triangle Theory (ASTT) — All angles in a triangle add up to 180°. A+B +C=180°.

Opposite Angle Theory (OAT) — Opposite angles are equal.

Supplementary Angle Theory (SAT) — Angles from slicing a straight line equals 180°.

Complementary Angle Theory (CAT) — If a 90° angle is is sliced, the angles have to add up to 90°.

Isosceles Triangle Theory (ITT) — The two angles that are opposite to the two equal side lengths, those angles are also equal.

Exterior Angle Theory (EAT) — The outside angle is equal to the inside opposite two angles added together.

Equilateral Triangle Theory (ETT) — In a equilateral triangle, all angles are 60°.

Sum of Exterior Angle Theory (SEAT) — The sum of a triangle's angles is 360°.

PARALLELOGRAM ANGLE PROPERTIES

F Pattern — Corresponding angles are equal.

Z Pattern — Alternate angles are equal.

C Pattern — Interior angles are supplementary therefore the interior angles add up to 180°.

POLYGONS

The sum of the exterior angles of a convex polygon is 360°.

For a polygon with *n* sides, the sum of the interior angles, in degrees, is 180(n - 2).

To calculate each interior angle of a polygon, the formula is: 180(n - 2) / n.

MIDPOINTS AND MEDIANS IN TRIANGLES

A line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.

The height of a triangle formed by joining the midpoints of two sides of a triangle is half the height of the original triangle.

The medians of a triangle bisect bisect the area.

MIDPOINTS AND DIAGONALS IN QUADRILATERALS

Joining the the midpoints of the sides of any quadrilateral produces a parallelogram.

The diagonals of a parallelogram bisect each other.

Chapter 8: Measurement Relationships

The formula for Pythagorean Theorem = $c^2 = a^2 + b^2$.
C is the hypotenuse.
A and B are the other sides.

AREA & PERIMITER FORMULAS (2D)
Square = S^2 — 4s
Rectangle = lw — 2(l+w)
Circle = πr^2 — πd
Triangle = bh / 2 — a+b+c
Trapezoid = (a+b)h / 2 — a+b+c+d
Parallelogram = bh — 2(b+c)

SURFACE AREA & VOLUME FORMULAS (3D)
Square-based Pyramid = $2bs+b^2$ — $b^2h / 3$
Triangular Prism = ah+bh+ch+bl — blh / 2
Rectangular Prism = 2(wh+lw+lh) — lwh
Cone = $\pi rs + \pi r^2$ — $\pi r^2h/3$
Sphere = $4\pi r^2$ — $4\pi r^3/3$
Cylinder = $2\pi r^2+2\pi rh$ — πr^2h

I do not take full credit for the work; information comes from various sources.
Use the notes at your own risk. I am not to blame if anything is incorrect or missing.