Equivalent Linear Relations and Systems:

Equivalent linear relations have infinite point of intersections.
Equivalent linear relations have the same graph.
You can can create an infinite amount of linear relations by multiplying/dividing the line by the same number.

One POI - Slopes and y-intercepts are different. $y = 5x + 7$, $y = 3x - 2$

Zero POI's - Slopes are the same, y-intercepts are different. $Y = 4x + 10$, $Y = 4x + 3$

Infinite POI's - Lines coincide. Slopes and y-intercepts are the same (can be equivalent).
$y = 3x + 5$, $2y = 6x + 10$

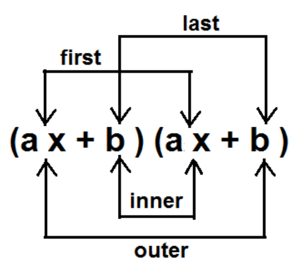
Equivalent linear systems have the same point of intersection.

Point of Intersection:
To find the POI without graphing the two lines, put both lines into $y = mx + b$ form and have them equal each other, then solve.
$y = -2x - 5$ and $y = 3x + 5$
$-2x - 5 = 3x + 5$
$x = -2$

Now substitute the x variable in either of the formulas, the answer is the y-cordinate.
$3 \times -2 + 5 = -1$

The POI of these two lines are: (-2,-1).

Foiling:
To find a binomial product, multiply the following:



An example...

$(x+4)(x+5)$
$= x^2+5x+4x+20$
$= x^2+9x+20$

TWO SPECIAL PRODUCTS

Perfect Square Trinomial — $(a+b)^2$

Difference of Squares — $(4a-2b)(4a+2b)$ [always a binomial]

FOILING QUICKLY (POWERS)

Foiling the "*fast way*" doesn't work on all expressions; it works on binomials being squared (^{2}).

1. Look at your expression —> $(m+4)^2$
2. Square the first variable.
3. Multiply the two terms by each other, then double the result.
4. Square the final term.
5. Done! —> $m^2+8m+16$

Factoring Checklist:

Factoring is the opposite of foiling.

1. Greatest Common Factor
- Greatest Common Factor
- Common Bracket
- Grouping Style

2. Trinomials
- Perfect Square
- Simple Trinomial
- Complex Trinomial

3. Difference of Squares

GREATEST COMMON FACTOR

- 2+ terms
- There must be a GCF between all terms
- The GCF can be a number, variable, or number and variable

WHAT TO DO

GCF (Divided all terms by GCF)

$12x^5+3$
$= 3(4x^5+1)$

$14m^6-7m^4+21m^2$
$= 7m^2(2m^4-m^2+3)$

Sometimes, there will not be a GCF among all terms, so you must group them together and then find a GCF between the terms you grouped together. From here, perform the grouping style.

m^2-4n+4m-mn	xy+12+4x+3y
= m(m+4)-n(4+m)	= y(x+3)+4(3+x)
= (m+4)(m-n)	= (x+3)(y+4)

PERFECT SQUARE

- 3 terms
- Can look like a simple or complex trinomial, but must pass the perfect square test

Perfect Square Test: The product of the square root of the first term and the square root of the last term, then doubled. If the answer = the middle term, it is a perfect square.

WHAT TO DO

(Square Root of the First Term -*+ Square Root of the Last Term)^2

-*+: The sign can be either positive or negative, follow the second term.

$16t^2$+24t+9	$3x^2$+6x+3
= $(4t+3)^2$	= 3(x^2+2x+1)
	= $3(x+1)^2$

SIMPLE TRINOMIAL

- 3 terms
- Numerical coefficient in front of the first squared term is 1

WHAT TO DO

Use the *product and sum theory* to help you factor. Find two numbers which products are the last term and its sum equal to the middle term.

x^2+8x+15	p^2-8p-20
= (x+5)(x+3)	= (p-10)(p+2)

COMPLEX TRINOMIAL

- 3 terms
- Numerical coefficient of the first squared term is not 1

WHAT TO DO

Use the decomposition method to factor. Find two numbers that give you the product of a and c and the sum of b.

$3m^2+10m+3$
$= 3m^2+9m+m+3$
$= 3m(m+3)+1(m+3)$
$= (m+3)(3m+1)$

$8y^2-22y+12$
$= 2(4y^2-11y+6)$
$= 2(4y^2-8y-3y+6)$
$= 2[4y(y-2)-3(y-2)]$
$= 2(y-2)(4y-3)$

DIFFERENCE OF SQUARES

- 2 terms
- The two terms must be separated by a minus sign
- You must be able to take the square root of both terms

WHAT TO DO

(Square Root of the First Term + Square Root of the Second Term)(Square Root of the First Term - Square Root of the Second Term)

x^2-16
$= (x+4)(x-4)$

$(x-4)^2-(x+3)^2$
$= (x-4+x+3)(x-4-x-3)$
$= (2x-1)(-7)$

Equation of a Circle:

When a circle is on the origin (0,0), the equation for it is $\mathbf{x^2+y^2=c^2}$.

To determine whether a point is on, inside, or outside of a circle; simply input the point's coordinates into the equation.

If the…
Left Side = Right Side —> The point is on the circle.
Left Side > Right Side —> The point is outside of the circle.
Left Side < Right Side —> The point is inside of the circle.

Midpoint of a Line Segment:

The Median: A line segment that cuts through the middle, but not at 90°.
The Altitude: Cuts at 90° not in the center. Altitude is the height.
The Right Bisector/Perpendicular Bisector: Cuts through the middle/midpoint and at 90°.

Midpoint formula (x, y)

$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Exponent Laws:

If multiplying, add the exponents. $x^m \cdot x^n = x^{m+n}$
If dividing, subtract the exponents. $x^m \div x^n = x^{m-n}$
If the exponents are separated by a bracket, multiply them. $(x^m)^n = x^{m \cdot n}$
If the variable/number is a fraction, multiply both the numerator and denominator. $(1/m)^n = 1n/mn$

Zero and Negative Exponents:

When the exponent = 0, the answer is 1 (or -1).
When the exponent is negative, find the reciprocal of the variable/number and turn the exponent into the opposite (positive). Then, apply the exponent to both the numerator and denominator.

The Trigonometric Ratios (when the triangle is right-angled):

SOH CAH TOA
Sin θ = Opposite / Hypotenuse
Cos θ = Adjacent / Hypotenuse
Tan θ = Opposite / Adjacent

Sine Law (when a side and the right angle opposite to that is known):

To find a side:

a / Sin A = b / Sin B = c / Sin C

To find an angle:

Sin A / a = Sin B / b = Sin C / c

Cosine Law (when two sides and the angle between them are known, or all three sides):

To find a side:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

To find an angle:

$\cos A = (b^2 + c^2 - a^2) / 2bc$

$\cos B = (a^2 + c2 - b^2) / 2ac$

$\cos C = (a^2 + b2 - c^2) / 2ab$

Completing the Square (Standard to Vertex):

Means to convert a non-perfect-square trinomial into a perfect square.

1. Find a number GCF of the first two terms (if none, use 1) and divide the first two terms by it, then leave the last term out the bracket pair
2. Divide the second term in the brackets by 2, then square it. Add it and subtract it within the brackets
3. Move the negative term outside the bracket, and multiply it by the a value
4. Factor the equation

The Descriptive Table:

A table which features a lot of information about a quadratic relation.

- Vertex — The (h,k) of the equation
- Direction of opening — Is the parabola concave up or down? A negative equation will be down, a positive will be up
- Axis of symmetry — The x coordinate of the vertex
- Minimum/maximum value and where it occurs — It is minimum if the equation is positive, and maximum if the equation is negative. Its value is the y coordinate of the vertex. To find out when it occurs, its simply the x coordinate of the vertex
- Domain — It is always $x \in \mathbb{R}$, meaning any set of real numbers
- Range — This is y≥**[y coordinate of the vertex]**. The greater than sign is a less than sign if the equation is negative.

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distributive Property:

The distributive property is to expand an expression and multiply all the terms in the brackets by the monomial term thats in front of the bracket. As an example, 5(x + 3) would become 5x + 15.

Substitution:

1. Expand both equations and write them in ax + by = c form.
2. Look and solve for the lonely variable. I.e: 3x + **y** = 7 —> y = 7 - 3x
3. Substitute the solution of the lonely variable into the opposite equation. If the lonely variable came from equation one, solve into two — vice versa.
4. Do the math and solve for x or y.
5. Find the other variable by substituting the variable you just found with the answer in either equations.

2x - 3y = 7
3x - **y** = 7

3x - 7 = y

2x - 3(3x - 7) = 7
2x - 9x + 21 = 7
-7x = -14
x = 2

3x - y = 7
3(2) - y = 7
6 - y = 7
y = -1

(4,5)

Elimination:

1. Expand both equations and write them in ax + by = c form.
2. Check if the equations have a +/- relationship.
3. If no relationship is apparent, create one by making an equivalent equation.
4. Do the appropriate operations between the two equations.
5. Find the other variable by substituting the variable you just found with the answer in either equations.

5m **+ 6n** = 15
-4m **- 6n** = -26
m = -11

5(-11) + 6n = 15
-55 + 6n = 15

6n = 15 + 55
6n = 70
n = 35 / 3

Length of a Line Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a Median:

ABC has vertices A(3,4), B(-5,2), and C(1,-4). Determine an equation for CD, the median from C to AB.
Finding the equation of the median:
1. Find midpoint of the line that the median line will be intersecting using the midpoint formula. **[Find M of ab — M ab = (-1 ,3)]**
2. Find the slope of the median line using the slope formula. **[Find m of CD — m cd = 7/-2]**
3. Find the equation of the median line using the equation formula. **[Y = -7/2x - 1/2]**

Extending/Shrinking Sides:

A playground, which measures 60m by 40m, is to be doubled in area by extending each side by an equal amount.

- Area needs to be doubled, so 2(60 x 40) = 4800m^2
- Now, use logic —> (60 + 2x)(40 + 2x) = 4800

If you get two roots, select the *one that makes sense with the question*

Forms:

$y = ax^2 + bx + c$ — Standard/Quadratic Function
$y = a(x - h)^2 + k$ — Vertex
$y = a(x - r)(x - s)$ — Intercept/Factored

Ball Problem:

The function $h=0.025d^2 + d$ models the height, h in meters, of one kick of a soccer ball as a function of the horizontal distance, d, in meters, from the place on the ground where the ball was kicked.

- To solve for the maximum height, complete the square
- To find a distance with a height, simply substitute h for that height and then perform BEDMAS backwards

Revenue:

(Number of units sold) x (Price per unit)

1. Layout the information; old units sold, old price
2. Find the new information; old units sold - x, old price + 5x
3. Put the new information into intercept form, then factor, then complete the square
4. You can now see your maximum revenue
5. To solve for the new price, input the x coordinate into the price equation you made previously

The Car Problem:

100 vehicles went through a car wash, a certain amount were trucks and cars. A wash costs $5 for cars, and $10 for trucks. The total amount earned was $600, how many trucks went through the car wash?

Let x represent cars
Let y represent trucks

$x + y = 100$
$5x + 10y = 600$

(just perform elimination or substitution)

$-5x + -5y = - 500$
$5x + 10y = 600$
$5y = 100$
$y = 20$

Therefore, 20 trucks went through the wash.

Suggested textbook review:

Pages 438 - 447

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